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A numerical study of vortex interaction

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Flow visualization has shown that the interaction of line vortices is a combination of tearing, elongation and rotation, the extent of each depending upon the flow conditions. A discrete-vortex model is used to study the interaction of two and three growing line vortices of different strengths and to assess the suitability of the method for such simulation.

Many of the features observed in experimental studies of shear layers are reproduced. The controlled study shows the importance and rapidity of the tearing process under certain conditions.

1. Introduction

Turbulent mixing results from the interaction of the large-scale structures – vortical structures – found in regions of mixing. An understanding of these interactions not only aids in the development of mathematical models for vortical flows but also in the control of such mixing which is pertinent to many engineering problems.

Experimental studies have revealed a coherent structure of the turbulent mixing layer. In fact the entanglement of such vortex structures dominates the full extent of the mixing. Subsequent numerical studies have supported (or been supported by) the results of experimental investigations.

The discrete-vortex method has been used in the simulation of such fluid motion. Unfortunately, the method has some inherent problems which have limited the application of the method. The discrete vortices used to model the vortex sheet have a tendency to random motion due to high induced velocities resulting from the close proximity of the discrete vortices. Such problems have been overcome in the present work by means of a rediscretization technique (Bromilow & Clements 1982) which has been shown to mitigate the problems. The reader should refer to the cited paper for a complete discussion of the technique and a comparison with other techniques.

This paper is concerned with the interaction of vortices with similar or different strengths. Such interactions have been discussed widely in the literature and different mechanisms have been proposed. It is intended to show that some of the interactions and mechanisms can be reproduced by a numerical simulation of this kind.

This study might indicate the suitability and conditions necessary for such a method to be a reliable tool of computational fluid dynamics. The limitations, as well as the successes, of any numerical technique must be borne in mind in any application.

2. Previous investigations – experimental and numerical

Flow visualization photographs have shown vortices interacting under various conditions. These results have created intrigue concerning the mechanism of the interactions thus leading to the development of various theories.

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Winant & Browand (1974) performed experiments at a moderate Reynolds number, which clearly showed three stages to the development of a vortex sheet: (i) wavelike structures; (ii) discrete vortices; (iii) rolling structures. The vortical structures tended to pair, roll around one another and amalgamate into a new vortex. This interaction resulted in a straining motion which caused the cross-section of the vortices to become ellipses with a major-to-minor axis ratio of around 2.

Experiments by Brown & Roshko (1974), at a higher Reynolds number, revealed that a vortex sheet possesses a very coherent structure. Eddy pairing was not as obvious, but on plotting the trajectories of the eddies the pairing and also triplet and quadruple amalgamation became evident. The interaction of the vortices provided a very definite mixing, as fluid from one side of the vortex sheet made deep incursions into the other side.

Rapid mixing was evident in the photographs of Dimotakis & Brown (1974). Entrained fluid was seen to remain relatively unmixed until the onset of amalgation, after which mixing was very rapid and accompanied by a tearing apart of the vortices.

Vortex interactions would seem to be a combination of several processes. Damms & Küchemann (1974) suggest that the processes are rotation, elongation and tearing, the first two being the most important. In contrast, an argument proposed by Moore & Saffman (1975), based on the stability of a vortex under the straining effect of other vortices, implies that tearing is the dominant process. The straining motion would cause vortices of circular cross-section to be replaced by elliptical vortices, a process that is supported by experimental evidence.

Apart from the obvious kind of interaction, the growth of subharmonics of the initial perturbation contributes to the development and subsequent breakdown of a vortex sheet. The work of Browand (1966) showed that the process of instability increased the number of important frequencies, from a single frequency, by the creation of higher and lower harmonics.

Numerical studies have resulted in some of the above observations being reproduced. Acton (1976) modelled an infinite thick vortex sheet (the Helmholtz instability) by four rows of point vortices. The vortex sheet was given a sinusoidal perturbation and the development followed.

Elongation of the vortex structures was observed, and also rotation. However, it was concluded by Acton that the tearing apart of vortex structures was far less likely to occur unless the array of vortices was very regular. Elongation and rotation were also simulated by the calculation of Ashurst (1977), which involved several thousand vortices (and 250 hours of computing). The tearing process was not particularly evident.

Christiansen & Zabusky (1973) used a vortex-in-cell appproach to study such interactions. Fusion of vortex structures together with elongation and rotation were observed in the development of the vortex sheet. However, on studying a collinear system of vortices, the linear-growth phase was followed by a period of rapid fission.

3. Foundations of the numerical method

Hama & Burke (1960) applied the discrete-vortex method to the development of a sinusoidally perturbed vortex sheet. They showed that a perturbation $y = \alpha \sin(2\pi x/\lambda)$ resulted in a vorticity distribution of the form

$$\gamma(x) = 2U\left(1 - \frac{2\pi\alpha}{\lambda}\cos\frac{2\pi x}{\lambda}\right),$$

where α is the amplitude of the disturbances, λ the wavelength and U the free-stream velocity. Since the total strength of the vortex sheet is $2U$, a sinusoidal perturbation results in a cosine vorticity distribution.

The development of this basic problem has been considered by many authors. One wavelength of the infinite vortex sheet is modelled by n discrete vortices. Differential equations for the components of induced velocity can be set up, integrated to provide new positions for the discrete vortices and so the calculation proceeds. A fourth-order Runge–Kutta integration scheme with a timestep of 0.025 was used in the present study, and either 36 or 42 vortices were used to model the sheet initially. The basic development is discussed by Bromilow & Clements (1982).

Certain conditions must be satisfied in order for an application of the discrete vortex method to be valid. The work of Moore (1979) shows that the initial-value problem for the Helmholtz instability has no solution for times greater than a critical time $t_c = O(\log E^{-1})$, where E is the initial amplitude of the disturbance. In the present work, the maximum amplitude is 0.02, thus $\log E^{-1}$ is 3.912. The interaction of the two vortices is stopped at a non-dimensional time of 0.70, and that of three vortices at 0.45. Thus one could argue that the time of calculation lies within the limits suggested by Moore (1979).

The same paper concludes that the vortex sheet is an inadequate approximation for a vortex layer unless it is everywhere undergoing rapid stretching. For the model reported, regions of rapid stretching are represented by discrete vortices, whereas in regions of compression an amalgamation technique is used.

Again, a real vortex layer of finite thickness can be represented by a vortex sheet only if the scale of motion greatly exceeds the vortex-layer thickness. Throughout this paper, the results of the simulation are compared with the experimental results of Brown & Roshko (1974). In order for such comparisons to be of value, it is necessary to argue that the vortex layer studied by Brown & Roshko can be represented by a vortex sheet. Such an argument is presented in the Appendix.

A study of interacting vortices is achieved by extending this basic formulation. By combining subharmonics of the fundamental perturbation, two or more vortices of different strengths will form in each wavelength of the perturbation. Of course, the overall disturbance is still periodic and this can limit the application of the model, as discussed later.

Thus a perturbation $y = \alpha \sin(4\pi x/\lambda)$ would result in two vortices per wavelength. By combining this perturbation with $\beta \cos(2\pi x/\lambda)$ (say), i.e.

$$y = \alpha \sin\frac{4\pi x}{\lambda} + \beta \cos\frac{2\pi x}{\lambda},$$

the two vortices will be of different strength. Using the work of Hama & Burke (1960), the vorticity distribution will be

$$\gamma(x) = 2U\left(1 - \frac{4\pi\alpha}{\lambda}\cos\frac{4\pi x}{\lambda} + \frac{2\pi\beta}{\lambda}\sin\frac{2\pi x}{\lambda}\right).$$

A knowledge of this distribution allows a discretization of the vortex sheet.

A similar technique was used for three interacting vortices, by combining $y = \alpha \sin(6\pi x/\lambda)$ with a subharmonic, i.e.

$$y = \alpha \sin\frac{6x}{\lambda} + \beta \sin\frac{4x}{\lambda}, \quad \text{or} \quad y = \alpha \sin\frac{6\pi x}{\lambda} + \beta \sin\frac{2\pi x}{\lambda}.$$

FIGURE 1. Differential stretching of the vortex sheet.

By varying the amplitudes α and β, vortices of different strengths can be obtained, and hence their interactions may be studied. The chosen values of α and β will be discussed in the relevant section.

The total perturbation was fixed and equal to 2% of the wavelength, i.e. $\alpha+\beta = 0.02$. This choice is quite arbitrary, but obviously too large a perturbation should be avoided. By fixing the value of the total perturbation and varying α and β within this constraint should reveal the effects, if any exist, of constituent parts of the disturbance.

The model also incorporates an amalgamation and rediscretization technique as discussed by Bromilow & Clements (1982).

4. The interaction of two vortices

The interaction of two vortices was achieved by combining a sine and cosine perturbation i.e.

$$y = \alpha \sin\frac{4\pi x}{\lambda}+\beta \sin\frac{2\pi x}{\lambda}.$$

By varying the values of α and β, vortices of different strengths can be obtained.

This paper concentrates on two cases:

$$\text{(i)}\ \alpha = 0.0175, \quad \beta = 0.0025 \quad \text{(figure 2)};$$

$$\text{(ii)}\ \alpha = 0.0075, \quad \beta = 0.0125 \quad \text{(figure 3)}.$$

Case (i) results in the formation of two vortices whose strengths vary only slightly, whereas in case (ii) the vortices have very different strengths. Other combinations of α and β were considered, intermediate to the above extremes, which confirmed the trends reported below.

As expected, the stronger vortex was always found to have a direct influence on the development of the weaker vortex. Discrete vortices were drawn away from the weaker part of the sheet. Thus, owing to the periodic form of the perturbation, the discrete vortices would move in the direction of the arrows shown in figure 1, so resulting in a concentration of vorticity at certain places. (N.B. the total vorticity contained in the actual amalgamated core is marked on some of the diagrams. The total vorticity in one wavelength is $2U$, i.e. 2 in these cases.)

The rediscretization technique was extended to allow a movement of vorticity between different parts of the sheet, but without reducing the detail of its description. Unlike other studies in which regions of stretching were soon lacking in vortices (and hence detail), the present vortex sheet was always modelled by an equispaced set of vortices.

Passive markers were used to show the differential stretching of the sheet, as illustrated in figures 2(a,b) and 3(a,b). The rate at which the markers move (and hence the transfer of vorticity) was found to depend upon their position and the difference between the strengths of the developing vortical structures. Thus for case (i) 50.5% of the total vorticity is in the first half-wavelength at non-dimensional time $t = 0$,

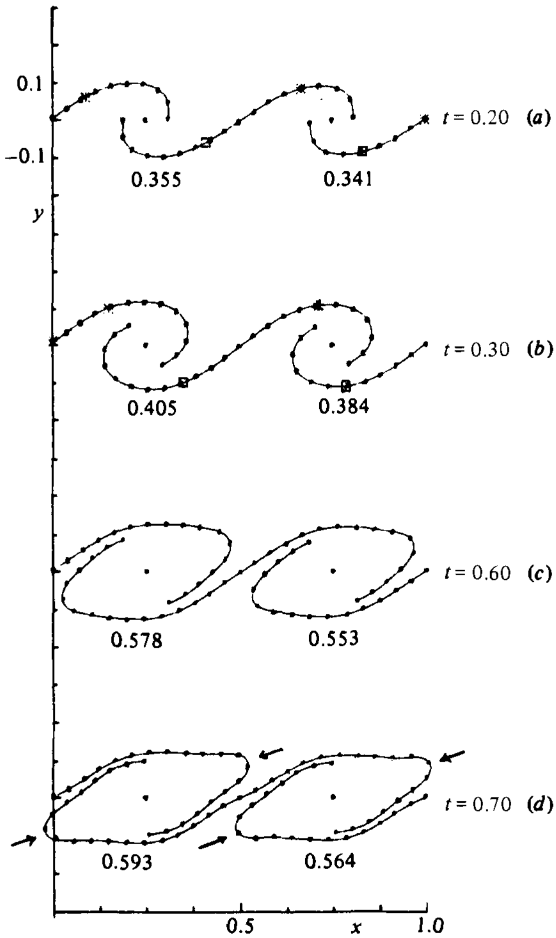

FIGURE 2. Interaction of two vortices.

and 52 % at $t = 0.8$; whereas for case (ii) there is 52.5 % initially, but 62.5 % at $t = 0.8$. In the obvious absence of rotation, this interaction is interpreted as the tearing process of Moore & Saffman (1978).

The calculation revealed a definite boundary between the fluid being drawn in one direction and that being drawn in the other. Once markers had come under the influence of either of the developing vortices, they would remain under such influence, and on reaching the amalgamation region would rotate in a clockwise direction. This and other similarities with the results of Acton (1976) support the replacement of clusters of vortices by an equivalent vortex surrounded by irrotational fluid.

The tearing process was found to be most significant in the early development of the vortex sheet, the time during which the rate of roll-up is greatest. Such behaviour is discernible from the vorticity distribution as parts of the sheet become very weak. Beyond such a time the circular vortices flatten and tend to ellipses (figures 2c, 3c), as reported by Winant & Browand (1974). The elliptical vortices in the results presented also possess a major-to-minor axis ratio of approximately two.

The amplitude of the disturbance or width H of the vortex sheet was found to reach a maximum, as illustrated in figure 4. When a large difference exists between the strengths of the developing vortices, the width of the vortex sheet is greater and the

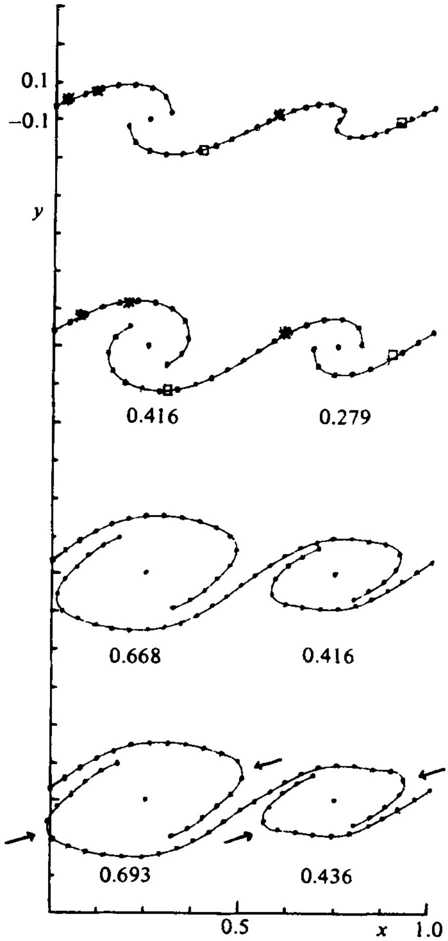

FIGURE 3. Interaction of two vortices.

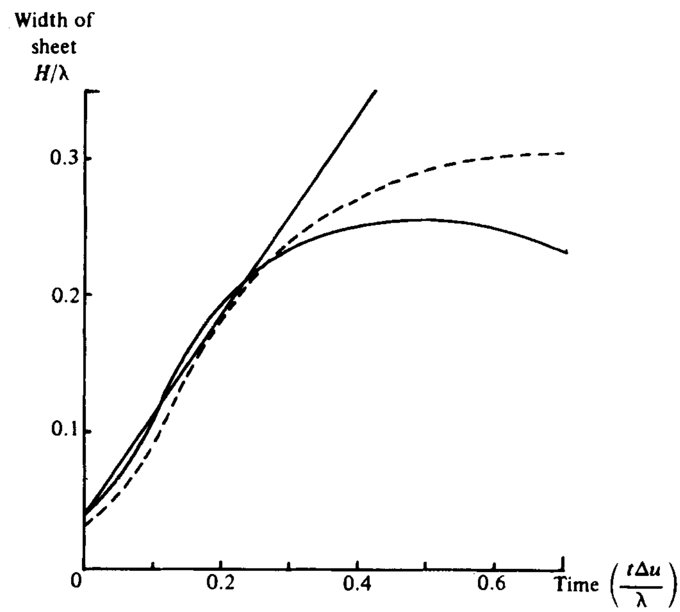

FIGURE 4. Spreading rate of vortex sheet (2 vortices): (——) case (i); (– – –) case (ii).

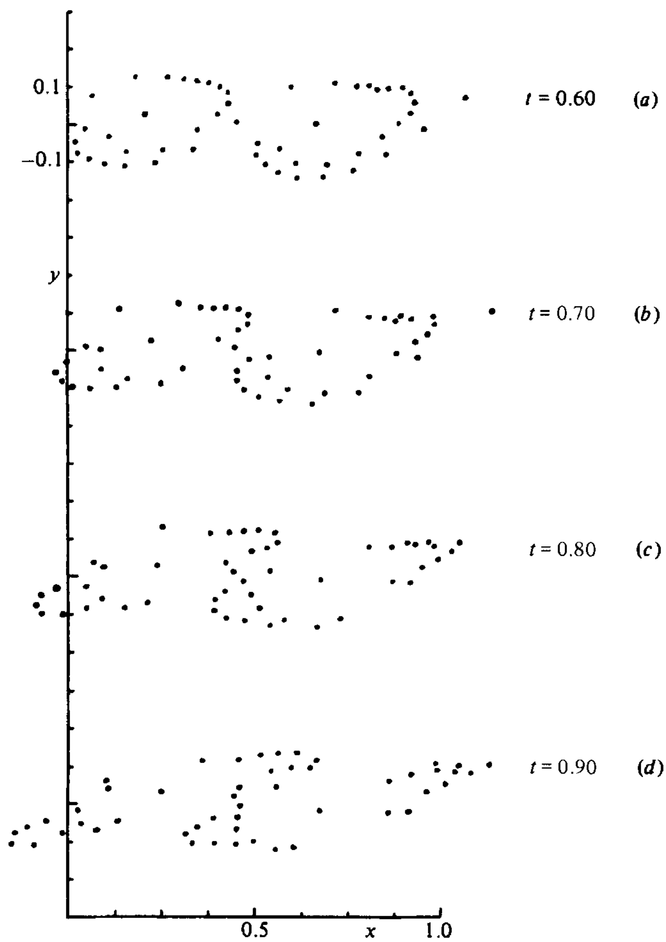

FIGURE 5. Interaction of two vortices – after relaxation of redistribution technique.

sheet thickens over a longer period of time. This occurs because a vortex is more able to draw off vorticity from another when there is a greater difference in their strengths. The straight line superimposed on figure 4 has gradient ≈ 0.37 (after scaling), which is the average spreading rate of a vortex sheet as proposed by Brown & Roshko (1974). The present vortex sheets spread at similar rates.

The width of the shear layer is seen to reach a maximum. This is to be expected, since the results show the interaction of two vortices only. As seen from the results of Brown & Roshko (1974), this initial interaction would be followed by further amalgamation and a spreading of the shear layer. The subsequent development would also require subharmonics of even lower orders to trigger it. The presented results therefore correspond to the first stage of vortex interaction.

The elongation of the vortical structures causes a thickening of the vortex sheet (figures 2d, 3d). This would normally facilitate the subsequent dissipation of vorticity by viscosity, especially in regions where the sheet is relatively weak. The straining motion responsible for the elongation is due to the developing very non-uniform distribution of vorticity, which results in secondary concentrations away from the regions of roll-up. These are indicated on figures 2(d) and 3(d) by arrows. Secondary concentrations were reported by Christiansen & Zabusky (1973), but the absence of the associated rotation, as reported by these authors, is probably due to the very regular initial arrangement.

In the simulations reported by other researchers, rotation was promoted by adding an initial asymmetry. Forcing of this kind did not lead to interaction by rotation in the reported calculations. However, 'switching off' the rediscretization process, after the formation of vortical structures, resulted in a rotation interaction. This is illustrated in figure 5.

Vortex interaction is thought to be responsible for turbulent mixing. In the results shown (figures $2d$), long arms of fluid from one side of the vortex sheet are seen to penetrate the other side. This would generate mixing of fluid in the manner suggested by the experimentalists.

At later times there are still major differences between the two systems of vortices. The stronger vortex retains its coherent structure longer than the weaker one – especially when the difference between the strengths is large. The dominating effect of the much stronger vortex enables the vortex sheet to grow over a longer period of time. With the absence of such a dominating vortex, localized instabilities soon become globalized, thus causing an overall breakdown in the coherent structure and so limiting the growth of the vortex sheet.

5. The interaction of three vortices

The development and interaction of three vortices resulted from the following combinations of subharmonics and fundamentals:

(i) $y = 0.012 \sin\dfrac{6\pi x}{\lambda} + 0.008 \sin\dfrac{4\pi x}{\lambda}$ (figure 6);

(ii) $y = 0.0075 \sin\dfrac{6\pi x}{\lambda} + 0.0125 \sin\dfrac{4\pi x}{\lambda}$ (figure 7);

(iii) $y = 0.015 \sin\dfrac{6\pi x}{\lambda} + 0.005 \sin\dfrac{2\pi x}{\lambda}$ (figure 8).

These three cases are reported since the interactions possess very distinct characteristics. Cases (i) and (ii) correspond to a strong–weak–strong situation with the difference in strengths much greater in case (ii). Case (iii) corresponds to a weak–strong–weak situation.

Unlike the case of two vortices, the present combinations of subharmonics have caused the centres of roll-up to be unevenly distributed within each wavelength. The positions of the two outer centres have been displaced towards the inner. As a result, the two outer vortices will have a greater influence on each other's development than would a corresponding vortex of the adjacent wavelength. A slight rotation of the whole wavelength has resulted which would probably be far greater if the initial arrangement was less regular.

Comparison between the development of two vortices and that of three is possible (table 1), but care must be taken in choosing the correct timesteps. If $T = Ut/\lambda$ is the non-dimensional time for the two-vortex case, then $T = Ut/\tfrac{2}{3}\lambda$ for the vortex case, i.e. the interaction of two vortices at time T should be compared with the interaction of three at time $\tfrac{2}{3}T$.

The onset of instabilities occurs much earlier during the interaction of three

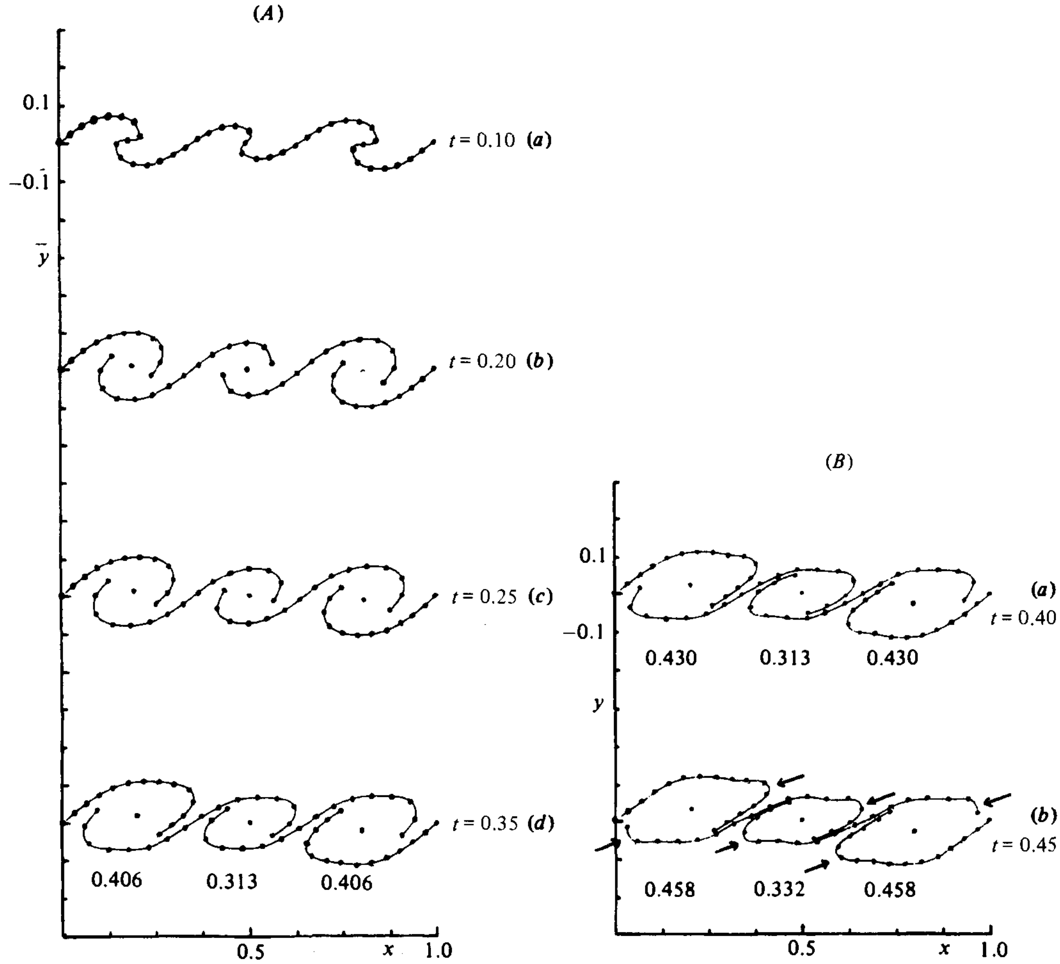

FIGURE 6. Interaction of three vortices.

vortices. The coherent structure of the mixing layer is destroyed sooner, thus reducing the growth of the perturbation and the extent of mixing. This is supported by the predictions of Moore & Saffman (1975), who set a stability criterion for arrays of interacting vortices. If l is the mean spacing of the vortices and δ the maximum thickness of the array, then vortices will interact destructively if $l < 0.35\delta$.

A weakness of the model emerges in case (i) (figure 6B). As a result of superposing subharmonics, the upper and lower sections of the vortex sheet about the first and third vortices possess different vorticity distributions. Consequently, part of the sheet is drawn towards a secondary concentration, and does not roll around the central vortex core as expected.

Apart from obvious similarities between the interaction of two and three vortices, the latter has resulted in more elaborate interactions. The strong–weak–strong case exhibits a definite tearing apart of the weaker vortex, especially in case (ii) (figure 7). Tearing is observed in case (i) (figure 6B), but there is more resemblance to the two-vortex case. This difference in behaviour is supported by the stability argument of Moore & Saffman (1975).

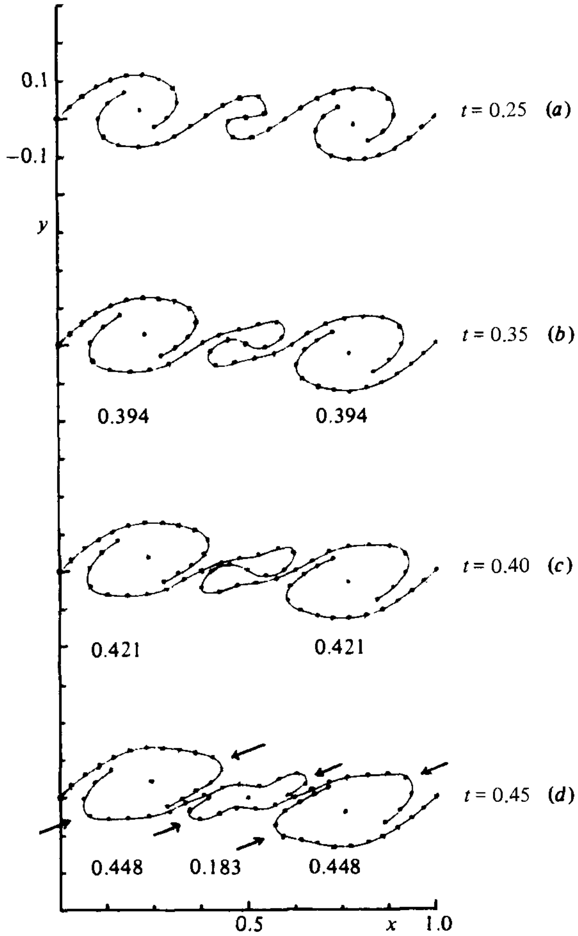

FIGURE 7. Interaction of three vortices.

Table 2 compares the interaction of three vortices (cases (i) and (ii)), and supports the above observations from the graphics by predictions using the Moore & Saffman criterion.

Finally, the weak–strong–weak case exhibits behaviour peculiar to itself. There is a one-sided elongation of the weaker vortex in the direction of the stronger (figures 8*c*, *d*). Elongation of this kind is a consequence of its interaction with the stronger vortex and also the weaker vortex of the adjacent wavelength.

The relative sizes of the vortex structures are, of course, dependent upon the amplitude of the underlying subharmonics. The subharmonic is strongest in case (ii), and here the spreading of the sheet is greatest. In case (iii) the middle vortex corresponds to the growing subharmonic.

6. Discussion and conclusion

Both advantages and disadvantages of the model have emerged from the present study. The initial conditions and extent of the flow are particularly important in the development of the vortex sheet, as demonstrated in this study, and hence results must be discussed in context.

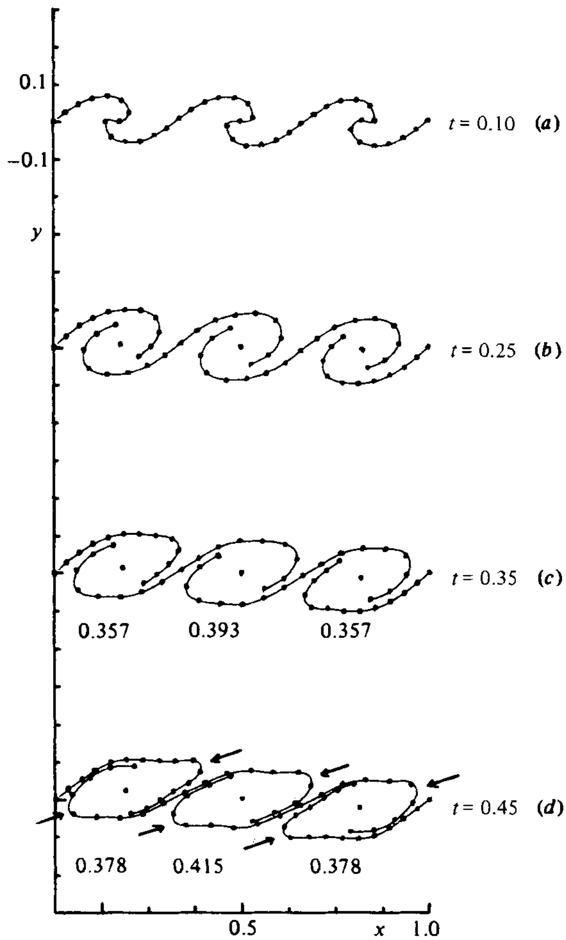

FIGURE 8. Interaction of three vortices.

No. of vortices	Time	l	δ	Prediction (Moore & Saffman 1975)	Interaction from graphics
2 (figure 2)	30	18.5	16	non-destructive	non-destructive
3 (figure 6*A*)	20	12.5	15.5	non-destructive	non-destructive
2 (figure 2)	60	7	17.5	non-destructive	non-destructive
3 (figure 6*B*)	40	3.5	15.	destructive	onset of tearing

TABLE 1. Comparison between the interaction of two and three vortices (rule for time comparison is used)

In the present study, the array of vortices was regular and thus reduced the possibility of amalgamation by rotation. This is unlike previous numerical studies of vortex sheets. Under such conditions, the only possible forms of interaction are tearing and elongation. Without redistribution and the coherent structure to the vortex sheet, rotation was found to dominate the interaction.

Use of the redistribution technique ensures that, in regions of rapid stretching, the coherent structure of the sheet is maintained, even though the vorticity becomes

	Time	l	δ	Prediction (Moore & Saffman 1975)	Integration from graphics
Case (i) (figure 6A)	0.25	10	16	non-destructive	non-destructive
Case (ii) (figure 7)	0.25	12	17	non-destructive	non-destructive
Case (i) (figure 6A)	0.35	6	15.5	non-destructive	non-destructive
Case (ii) (figure 7)	0.35	5	17.5	destructive	onset of tearing

TABLE 2. A comparison between the development of three vortices

weak. When no such technique is used, discrete vortices in regions of stretching are rapidly drawn apart, leaving two or three regions of vortex clusters with few discrete vortices between them. This suggests that the 'continuous' structure of the sheet must be destroyed before vortex pairing can occur. In the simulation of purely rotational interaction as performed by Acton (1976), a similar breakdown occurred before the onset of pairing.

Even with the inclusion of redistribution, rotation would probably dominate the interaction if the calculations were continued for longer times. But, as shown by Maskew (1977), significant errors occur in calculations of this kind when neighbouring sheets are much closer than the point-vortex separations. As seen in some of the figures, this point would soon be reached unless more vortices were included in the calculations. However, whether the increase in computer costs is worthwhile is questionable, since at late times the development would be dominated by viscous effects.

The extent of the tearing process has been controlled by the proximity of the vortices, as proposed by Moore & Saffman. A lack of rapid tearing was replaced by elongation and an overall thickening of the vortex sheet.

Thus in real flows, where the arrays of vortices would be more irregular, a combination of the three processes would be expected, but the extent of each process depending upon the structure of the sheet. The rapidity of the tearing process would indicate it to have a dominant effect, but only under certain circumstances, as seen in the present study.

The extent of the coherent structure of the vortex sheet was observed to depend on the proximity of vortices and the relative strengths. Such conditions are related to the importance of subharmonics present in the initial perturbation. The higher the number of important subharmonics in the wavelength, the earlier the breakdown of the coherent structure. On the other hand, fewer important subharmonics permitted an extended growth of the sheet. In real flows, the growth of the sheet might well extend, as a more complicated array of vortices would promote further growth of the stronger vortices.

The elongation of the vortices resulted in the development of secondary vortices, which have been clearly observed in experimental and numerical studies. Their importance in the present study would suggest that these vortices might control the later development of the sheet, once the initial structure has been destroyed. This supports the possibility of a cascading process from larger to smaller vortices.

As a check on the accuracy of the discrete-vortex method, van de Vooren (1980) suggested that the Hamiltonian and vorticity centroid could be used as invariants. Since the set of discrete vortices is changing at each timestep, the Hamiltonian, in its usual form, is of no significance. For the interaction of two vortices, the vorticity centroid changed by a maximum of 0.5% over twenty steps. The corresponding change for three vortices in 0.6%.

Overall, the study has provided support for certain suppositions and identification with accepted results. The advantage of the present method over previous ones is the ease at which a controlled study can be made, in order to provide more understanding of vortex dynamics.

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Appendix. The applicability of the results of Brown & Roshko (1974) to a vortex sheet

In order for a valid comparison to be made between the experimental results of Brown & Roshko (1974) (for a vortex layer of finite thickness) and those of the presented simulation, it is necessary to show that, in the experimental study, the scale of motion exceeded the vortex-layer thickness. Thus, it is required to estimate the thickness of the shear layer in the shadowgraph plates of Brown & Roshko.

It may be shown that a solution to the boundary-layer equations applied to the problem of the growth of the interfacial layer occurring when two parallel flows of velocities U_1 and U_2 meet at $x = 0$ has a similarity solution with

$$\begin{aligned} \frac{u}{U_1} &= f\left[y\left(\frac{U_1}{\nu_1 x}\right)^{\frac{1}{2}}\right] \quad (y > 0) \quad (U_1 > U_2), \\ &= f\left[y\left(\frac{U_1}{\nu_2 x}\right)^{\frac{1}{2}}\right] \quad (y < 0). \end{aligned}$$

The form of the function f has to be determined numerically, and is dependent on the velocity ratio $r = U_2/U_1$. The form of f for $r = 0.5$ and $r = 0$ is given by Batchelor (1967). The solution can further be extended to flows in which $\rho_1 \neq \rho_2$ and $\nu_1 \neq \nu_2$. Profiles for $r = 0$, $\rho_2^2\nu_2/\rho_1^2\nu_1 = 10$, 100, 5.97×10^4 are also given by Batchelor.

μ is sensibly constant for air, N_2 and He in the range 1–9 atm used by Brown & Roshko. Reference to Kaye & Laby (1975) shows that the viscosities change by $\sim 1.5\%$, $\sim 1.1\%$ and $\sim 0\%$ for change of pressure from 1 to 23 atm. Kaye & Laby give

$$\begin{array}{cccc} & \text{air} & \mathrm{N_2} & \text{He} \\ \dfrac{\mu}{\mu\mathrm{N\,s\,m^{-2}}} = & 18.2 & 17.6 & 19.6. \end{array}$$

ρ is proportional to pressure, so $\nu(=\mu/\rho)$ is inversely proportional to pressure. Using $\rho = 1.196$ kg m^{-3} for dry air at 20 °C and 1 atm, and using the figures of Brown & Roshko,

$$\frac{\rho_{\text{air}}}{\rho_{\mathrm{N_2}}} = \tfrac{29}{28}, \quad \frac{\rho_{\text{He}}}{\rho_{\mathrm{N_2}}} = \tfrac{1}{7},$$

we have

	air	N_2	He
$\dfrac{\rho}{\mathrm{kg\ m^{-3}}}$ =	1.196	1.155	0.165.

Then $\nu/10^{-6}\ \mathrm{m^2\ s^{-1}}$ is given as follows:

	air	N_2	He
1 atm	15.217	15.238	118.788
4 atm	3.804	3.810	29.697
7 atm	2.174	2.177	16.970

The length of the shadowgraph pictures can be inferred to be ~ 0.16 m (scale given in figure 3 of Brown & Roshko), so we might expect a flat transition layer with conditions as figure 5 (of the same paper) to have

$$y_{\text{upper}} \approx 4\left(\frac{3.810\times 0.16\times 10^{-6}}{8.76}\right)^{\frac{1}{2}} = 1.05\times 10^{-3}\ \mathrm{m},$$

$$y_{\text{lower}} \approx -6\left(\frac{29.697\times 0.16\times 10^{-6}}{3.30}\right)^{\frac{1}{2}} = 7.2\times 10^{-3}\ \mathrm{m}.$$

Now the boundary-layer momentum thickness above and below the dividing plane at the point where the shear layers meet is estimated by Brown & Roshko to be 0.025×10^{-3} m (10^{-3} in.). Twice this boundary-layer thickness could pessimistically be added to the transition-layer thickness.

The above has assumed that the transition layer remains flat. It does not do so of course, and the manner in which it does not do so is the whole point of Brown & Roshko's study. The transition layer rolls up into large vortices. The rapid stretching of the layer required to achieve this rolling-up may intuitively be expected to thin it, though the additional length could be taken as indicating that a larger value of x than merely the streamwise coordinate should be used in estimating the transition-layer thickness. Since this is an order-of-magnitude estimate, it is assumed (probably on the flimsiest of intuition) that these two effects cancel out.

Hence a crude estimate of the vortex-layer thickness developed by the right-hand edge of figure 5 of Brown & Roshko is 8.3 mm in a mixing-layer width of $\sim$ 30 mm. A similar estimate for their figure 4 yields a vortex-layer thickness of 4.6 mm and for their figure 6 (a) 5.52 mm.

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